



Progression from Mental Methods Towards a Standard Written Method of Calculation

STATEMENT OF INTENT

1.1 This policy is aligned with the National Curriculum objectives for Mathematics (2014). It provides guidance on effective written strategies for calculation and gives year-by-year expectations of a range of methods set out under the following headings: addition and subtraction strategies, multiplication and division strategies.

1.2 This policy draws on the Southwark Mental and Written Calculation Strategies documents and Maths Mastery concepts.

1.3 Mathematical understanding is developed through the use of representations that are first of all **concrete** (e.g. Dienes, apparatus), then **pictorial** (e.g. array, place value diagrams) to then facilitate **abstract** working (e.g. mental methods, columnar addition, long multiplication). Mental methods should be revised and used alongside written methods.

1.4 Our aim is that pupils will use mental methods (including with the use of jottings/informal recording) as their first port of call, when appropriate. However, for calculations that they cannot do mentally, they will need to use an efficient written method accurately and with confidence (including with missing digits to develop fluency.)

1.5 This policy is set out to ensure **consistency in progression** throughout the school and encourage tracking back and forwards depending on a pupils' **conceptual understanding**, their **mathematical skills** and their **knowledge** and **use of facts and vocabulary**.

USING RIVERSIDE'S CALCULATION POLICY

2.1 The strategies that pupils should be aware of are detailed for each year. Pupils should be encouraged to use a range of strategies and to consider the most *appropriate* method for any given calculation. Pupils' ability to use a range of strategies will be given greater emphasis than just relying on a formal written method.

2.2 The models and images included facilitate access to strategies and can be used to support pupils in explaining their reasoning. They are a visual and concrete image to support teachers' explanations for any given strategy. Pupils can then use these models to support their thinking and aid calculation. Pupils will be encouraged to become as efficient as possible in their jottings until they no longer need to record each step to support their thinking. Suggested images and materials are examples only and pupils will be exposed to a **variety** of different images and materials to aid fluency. The bar model will be used where appropriate to show both inverse relationships and to support missing digit calculations.

2.3 It is important that teachers are aware of pupils' strategies and their fluency when diagnosing their level of competency in calculation, as well as considering whether the answer is correct.

SECTION A

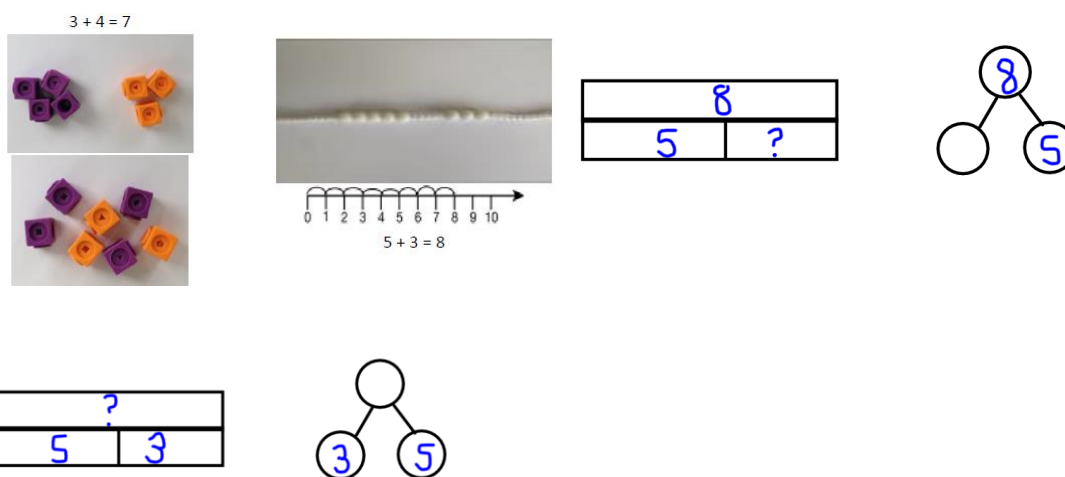
ADDITION AND SUBTRACTION STRATEGIES (BY YEAR GROUP)

Year 1

Addition and subtraction will be taught alongside each other to develop confidence and fluency with whole numbers. In Year 1, pupils begin to develop their mental strategies. These will still be aided with concrete and pictorial representations. Pupils need lots of experience partitioning numbers within 10 in different ways. This will aid mental addition and subtraction, particularly when bridging ten.

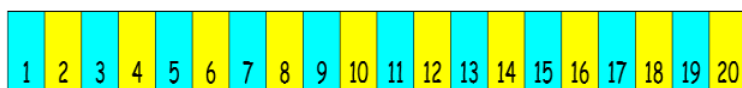
3.1 Joining two groups and then recounting all objects using one-to-one correspondence (without bridging ten)

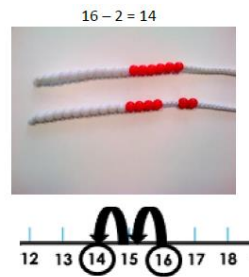
As with all strategies, pupils will start with concrete materials. With the bar model, cubes could then be joined to represent the bar and link the concrete and pictorial representations:



3.2 Counting on and back in ones from any given number

Pupils will count on or back in ones, from 0, 1 or any number first using resources then with the use of a marked number line and/ or a number track:



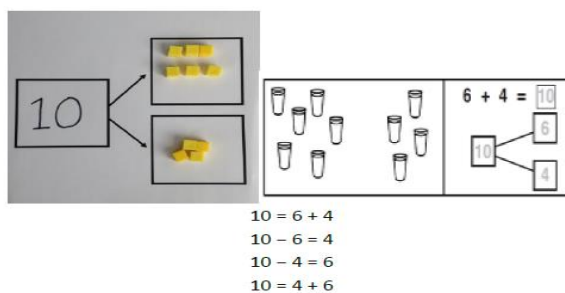


This should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that **counting on from the greater is more efficient.**

3.3 Number bonds within and to ten using the concept of part-part-whole

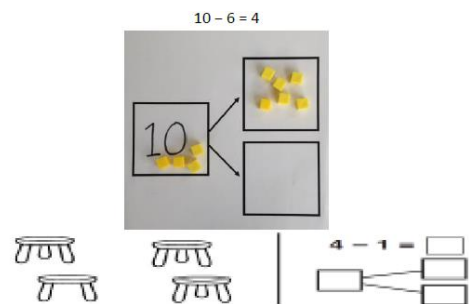
Addition and subtraction to be taught alongside each other as pupils will use a range of models to identify the inverse link between them.

The model below begins to develop the understanding of the commutativity of addition as pupils become aware that the parts will make the whole in any order. They will also become aware that this does not apply to subtraction.



?	
2	8

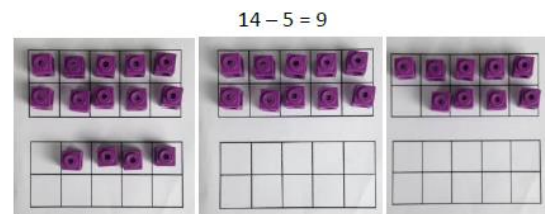
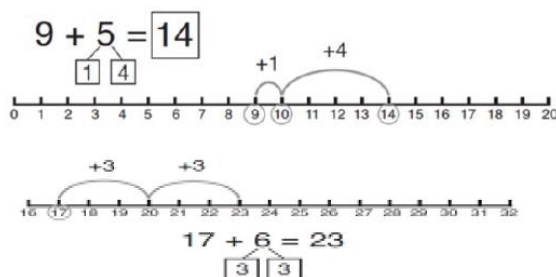
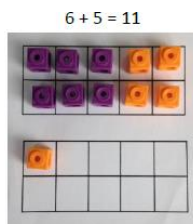
10	
?	8



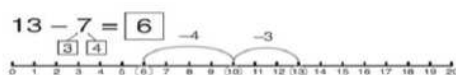
3.4 Adding and subtracting by make ten (using knowledge of bonds to 10)

Pupils will be encouraged to start at the bigger number and use the smaller number to make ten.

The colours of the beads on the bead string make it clear how many more need to be added to make ten. In addition, the empty spaces on the ten frame make it clear how many more are needed to make ten.



$$14 - 4 - 1 = 9$$



3.5 Adding using recall of doubles

Pupils will use their knowledge of doubles to add: $5 + 5$ is double 5

Pupils will begin to use their knowledge of doubles to add near doubles: $5 + 6$ is double 5 add 1

3.6 Using related calculations

Pupils will use knowledge of place value and related calculations: $17 + 3 = 20$ using $7 + 3 = 10$

Pupils will use their understanding of the relationship between addition and subtraction and that addition can be done in any order, using resources to support understanding:

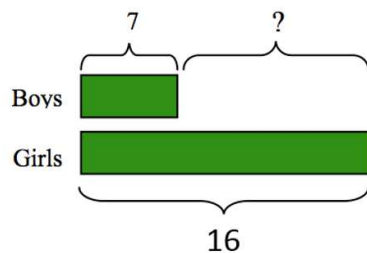
$$\begin{array}{ll} 7 + 2 = 9 & 2 + 7 = 9 \\ 9 - 2 = 7 & 9 - 7 = 2 \\ 17 + 2 = 19 & 2 + 17 = 19 \\ 19 - 2 = 17 & 19 - 17 = 2 \end{array}$$

This should lead to missing number calculations:

If $7 + 2 = 9$ then $17 + ? = 19$.

Year 2

After reviewing number bonds to ten and within ten using concrete materials in Year 1, pupils will then become fluent in their recall of addition number bonds to ten and their corresponding subtraction facts. They explore different ways of making twenty so that by the end of the year, they can recall and use facts to 20 mentally. When teaching addition, ensure pupils understand and use the commutative law.

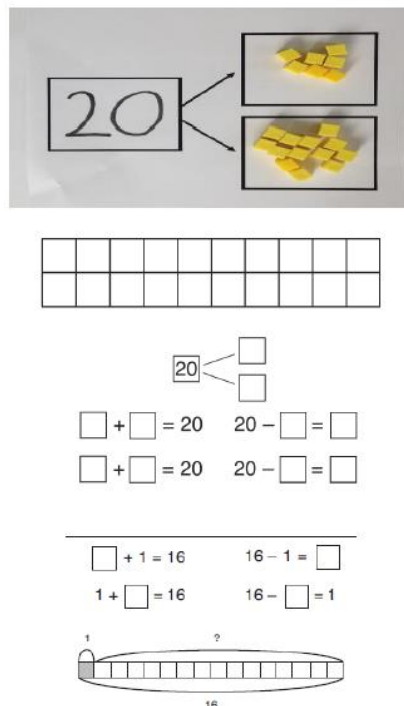


The bar method can be used throughout to develop pupils' understanding of addition and subtraction as inverse operations.

Throughout the year, pupils will continue adding numbers using their knowledge of number bonds to 10 and 20, e.g. $17 + 6 = 17 + 3 + 3$.

4.1 Number bonds to and within twenty using concept of part-part-whole

(See Year 1 guidance)



4.2 Partition, add and recombine

Pupils will use their understanding of place value to partition numbers into tens and ones:

$$30 + 2 = 32$$

$$32 - 2 = 30$$



Pupils will begin using two-digit and one-digit numbers not bridging ten. They should be confident in comparing numbers using place value, for example in $36 - 5$, 6 is more than 5 so the tens digit will not change. This means they can use their known facts $6 - 5$.

When bridging to the next ten, pupils will partition the one-digit number to 'make ten.' This is why their quick recall of number bonds to ten is **crucial**. Resources such as Dienes and ten frames will be used alongside to ensure conceptual understanding.

$$36 + 5 =$$

$$36 + 4 + 1 = 41$$

and

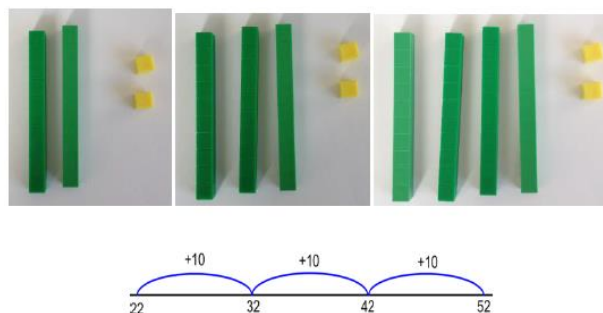
$$44 - 6 =$$

$$44 - 4 - 2 = 38$$

In order for pupils to successfully add or subtract multiples from any two-digit number, pupils first need to be able to confidently add and subtract two multiples of ten

(e.g. $40 - 20$, $30 + 40$, $30 - ? = 10$)

4.3 Count on in tens and ones from any number



4.4 Adding and subtracting multiples of ten

Before pupils can add two 2-digit numbers, they need to be confident with adding and subtracting multiples of ten, e.g. $34 + 20$

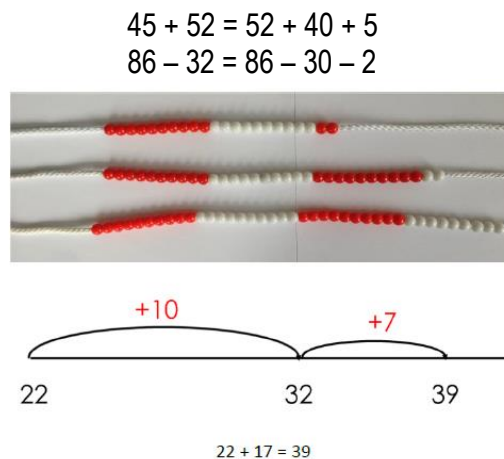
4.5 Using related facts for multiples of ten

Pupils will begin to use their knowledge of known facts. Dienes blocks can be used alongside pictorial and abstract representations to embed concepts.

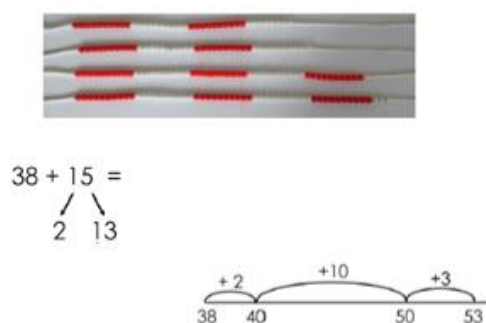
$$\begin{array}{ccc} \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} + \begin{array}{c} \cdot \\ \cdot \\ \cdot \end{array} & = & \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} & 3 + 4 = 7 \\ \text{leads to} & & & \\ \begin{array}{c} ||| \\ ||| \\ ||| \end{array} + \begin{array}{c} |||| \\ |||| \\ |||| \end{array} & = & \begin{array}{c} ||||| \\ ||||| \\ ||||| \\ ||||| \end{array} & 30 + 40 = 70 \end{array}$$

e.g. when calculating $52 - 40$, pupils can use their knowledge of $5 - 4$, linking to $50 - 40$ ($52 - 40$. $50 - 40 = 10$. $10 + 2 = 12$.)

4.6 Partitioning the smaller number in addition and the second number in subtraction

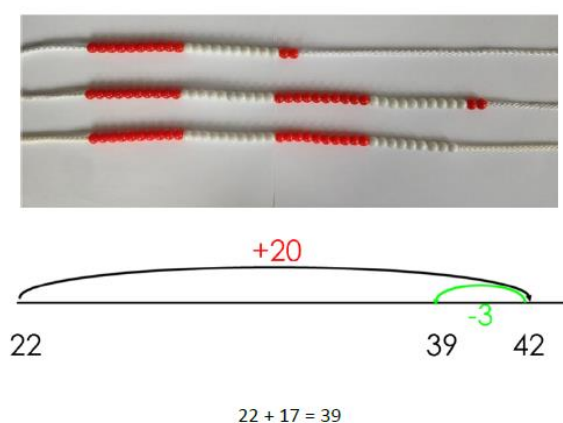


4.7 Make ten (using knowledge of bonds to 10) to add two two-digit numbers



Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.


4.8 Adding near multiples of 10 (by rounding one number, then adding the tens and taking away extra ones)



4.9 Adding three single digit numbers (make ten first)

The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.

The second bead string shows 4, 6 and then 7. The final bead string shows how they have now been put together to find the total.



$$\begin{array}{r} 4 + 7 + 6 = 10 + 7 \\ \quad \quad \quad 10 \quad \quad \quad = 17 \end{array}$$

4.10 Begin to use knowledge of place value to arrange numbers into column (preparing for column addition and subtraction in Year 3)

$$24 + 17$$

Tens	Ones
2 tens	4 ones
1 ten	7 ones

Tens	Ones
2 tens	4 ones
1 ten	7 ones

Tens	Ones
2 tens	4 ones
1 ten	7 ones

Tens	Ones
2 tens	4 ones
1 ten	7 ones

Tens	Ones
2	4
+	1
1	7
<hr/>	
4	1

$$34 - 13 = 21$$

Tens	Ones
3 tens	4 ones

Tens	Ones
3 tens	4 ones

Tens	Ones
3	4
-	1
1	3
<hr/>	
2	1

Year 3

In Year 3, pupils build on efficient methods introduced in Year 2 and apply these mentally. Resources and images from Year 2 can still be used to support children with the mental strategies below. Pupils will look at the digits within the calculation and use their knowledge of place value and number bonds to select the most appropriate method (e.g. no regrouping, so mental method using partitioning.)

5.1 Mentally add and subtract two-digit numbers; the answers may exceed 100

$$\begin{array}{l}
 78 + 26 \\
 \swarrow \searrow \\
 98 + 6 = 98 + 2 + 4 = 104
 \end{array}
 \qquad
 \begin{array}{l}
 78 - 29 \\
 \swarrow \searrow \\
 58 - 9 = 58 - 8 - 1 = 49
 \end{array}$$

Visual images such as the bar method will be used alongside to support understanding of the inverse and to model missing digit calculations:

<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; width: 50%; padding: 2px 10px;">78</td> <td style="border: 1px solid black; width: 50%; padding: 2px 10px;">26</td> </tr> <tr> <td colspan="2" style="border: 1px solid black; padding: 2px 10px;">?</td> </tr> </table>	78	26	?		<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; width: 50%; padding: 2px 10px;">78</td> <td style="border: 1px solid black; width: 50%; padding: 2px 10px;">?</td> </tr> <tr> <td colspan="2" style="border: 1px solid black; padding: 2px 10px;">104</td> </tr> </table>	78	?	104	
78	26								
?									
78	?								
104									
$? = 78 + 26$ $? - 78 = 26$	$104 - 78 = ?$ $104 - ? = 78$								

5.2 Add and subtract three-digit numbers and ones mentally

$386 + 3$ using known facts when there is no regrouping ($6+3$)

$386 + 8 = 386 + 4 + 4$ (making a multiple of 10)

$787 - 8 = 787 - 7 - 1$ (making a multiple of 10)

5.3 Add and subtract three-digit numbers and multiples of ten and one hundred mentally

At this stage, pupils will be very confident with partitioning and place value, applying this knowledge fluently within their mental strategies. Initially, this will be modelled using concrete materials such as Dienes and place value counters.

Using their knowledge of place value, pupils will notice that this will not bridge to the next hundred:

$$456 + 30$$

$$5 + 3 = 8 \text{ therefore } 50 + 30 = 80$$



The same will be applied to subtraction (e.g. $456 - 30$)

For calculations where regrouping is required, pupils can use varied strategies (see guidance from Year 2: commutative law, partitioning effectively) and the focus will be on accuracy and efficiency.

$$457 + 60 =$$

$$50 + 50 + 10 = 110$$

$$400 + 110 + 7$$

$$456 - 60$$

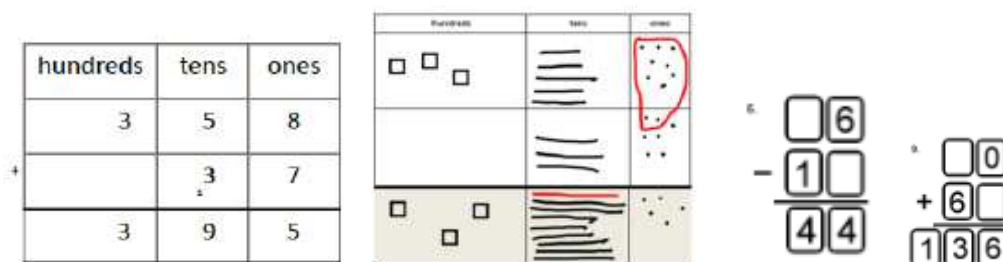
$$456 - 50 - 10$$

$$456 - 60$$

$$460 - 60 + 4$$

5.4 Add and subtract numbers with up to three-digit numbers using column method

Dienes blocks and/or place value counters will be used alongside the pictorial representations during direct teaching and there will be a focus on the language of hundreds, tens and ones to support conceptual understanding (e.g. 8 ones plus 7 ones are equal to 15 ones. 15 ones are equal to 5 ones and one ten.) Pupils will begin to recognise that this method is efficient for calculations requiring more than one element of regrouping. Pupils will be encouraged to use mental methods if there is no regrouping. Discussions will be had when using column method about when it is appropriate.



Year 4

6.1 Add and subtract numbers up to 1000 mentally

If no regrouping or regrouping in one instance is required, pupils will decide if mental strategies covered in previous years could be used efficiently or whether column method would be more appropriate.

$$135 + 614 = 614 + 100 + 30 + 5 = 614 + 130 + 5$$

$$815 - 309 = 815 - 300 - 9 = 815 - 9 - 300$$

$$127 + 63 = 127 + 3 + 60 = 130 + 60$$

$$488 + 15 = 488 + 12 + 3$$

$$335 + 337 = 335 + 335 + 2 \text{ (adding near doubles)}$$

Rounding and readjusting

When looking at mental methods, rounding and readjusting strategies will be taught when one of the numbers is close to a multiple of ten:

$$815 - 309 = 815 - 310 + 1$$

$$651 - 135 = 651 - 140 + 5$$

6.2 Start adding simple decimals in the context of money and measures

Pupils will use their understanding of the place value of pounds, pence and measures to add and subtract numbers with up to one decimal place mentally (where no regrouping is required.) In the context of money, zero as a place holder will be recorded.

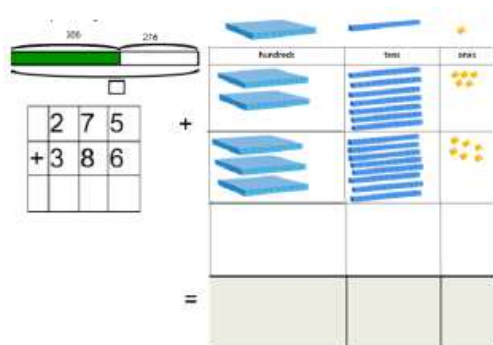
$$£2.40 + £0.50 = £2.00 + £0.40 + £0.50 = £2.00 + £0.90$$

$$£3.60 + £1.10 = £3.60 + £1.00 + £0.10$$

6.3 Add and subtract numbers with up to three-digit numbers using the column method

It is vital that pupils are made aware that this is **one possible strategy** and that they must decide which is the most efficient to make an accurate calculation. Direct teaching of the columnar method will require **at least one element of regrouping**, so that pupils are clear about when it is most useful to use it.

When using Dienes or other concrete resources to model subtraction, the number being subtracted should be written rather than being represented by dienes to avoid confusion.



$$\begin{array}{r}
 \begin{array}{|c|c|c|} \hline \text{hundreds} & \text{tens} & \text{ones} \\ \hline \square & \text{---} & \text{---} \\ \hline \end{array} & & \\
 - & & \\
 \hline
 & 1 & 8 \\
 \hline
 1 & 2 & 9
 \end{array}$$

hundreds	tens	ones
1	4	7
-	1	8
<hr/>		
1	2	9

5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one.

2	3	6	
+	4		4
<hr/>			
	8	1	2

1	3			
-				
<hr/>				
	5	3		

2	4	6	
-			
<hr/>			
	6	2	

3			9
-			
<hr/>			
	3	1	

Year 5

Pupils will be confident in counting on and back in tens of thousands, thousands, hundreds and tens to numbers greater than 1000 and then use this to support mental calculations. These will build on strategies from previous years, now applying to increasingly larger numbers using knowledge of place value. Pupils recognise that they can add or subtract what they have partitioned in any order. Pupils will be able to select efficient written and mental methods for calculations within increasingly complex numbers and problems.

$$61,450 - 30,000 \text{ (place value- known facts)}$$

$$1960 + 200 \text{ (place value with regrouping)}$$

$$12,462 - 2,300 = 12,462 - 2,000 - 300 = 10,462 - 300 \text{ (partitioning using place value)}$$

$$456,392 - 5,200 = 456,392 - 200 - 5,000 = 456,192 - 5,000 \text{ (partitioning)}$$

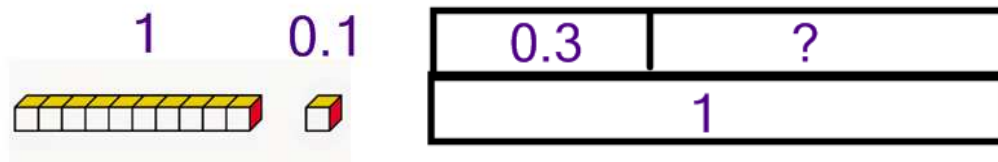
$$1936 - 740 = 1940 - 740 - 4 \text{ (rounding and adjusting)}$$

$$1250 + 268 = 1250 + 250 + 18 \text{ (using knowledge of near doubles)}$$

7.1 Use addition and subtraction facts to 10 and relate to tenths when adding and subtracting tenths (including compliments to 1)

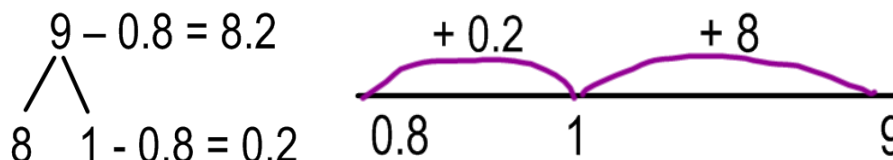
Resources will be used to conceptualise tenths as a proportion of a whole using materials such as Dienes and place value counters (e.g. a tens rod representing one whole.) Pupils will move to mental strategies and not become reliant on materials.

$$8 + 2 = 10, \text{ therefore } 0.8 + 0.2 = 1$$



7.2 Mentally add and subtract one digit whole numbers and tenths

$$5 + 0.2$$



7.3 Use addition and subtraction facts to 100 and relate to hundredths when adding and subtracting hundredths (including compliments to 1)

$$83 + 17 = 100 \text{ therefore } 0.83 + 0.17 = 1$$

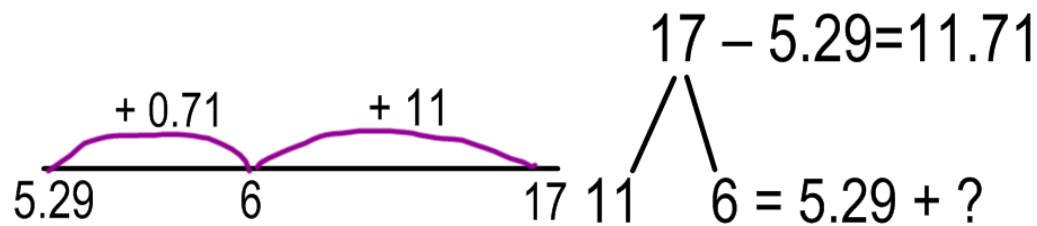
?	0.83
1	

7.4 Add and subtract whole numbers and numbers with two decimal places

$$3 + 0.56$$

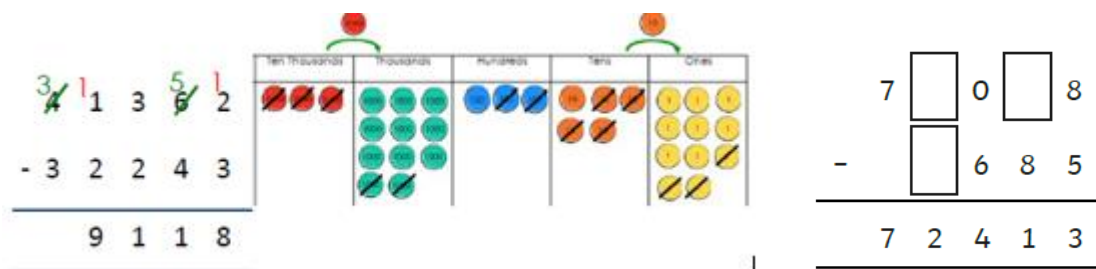
$$17 + 5.29 = 17 + 5 + 0.29$$

$$17 - 5.29$$



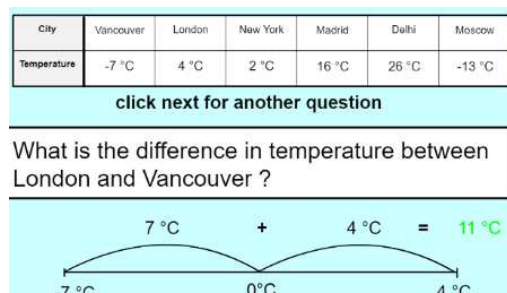
7.5 Add and subtract numbers with more than four digits using the columnar method

Images can be used to support understanding, especially with the place value of larger numbers, but will not be relied upon by pupils.



7.6 Count forwards and backwards through zero to include negative numbers

Pupils will extend the number line past zero and recognise the importance of zero in bridging positive and negative numbers.



Year 6

Pupils will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems (see previous year's guidance for methods.)

Our aim is that by the end of Y6, pupils use mental methods (with jottings) when appropriate for more complex calculations, including mixed operations with increasingly large numbers or decimals. However, for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.

$$1488 + 165 + 12 = 1488 + 12 + 165 \text{ (reordering to get a multiple of ten)}$$

$$4.8 + 2.5 - 1.8 = 4.8 - 1.8 + 2.5$$

$$8,250 - 998 = 8,250 - 1,000 + 2 \text{ (rounding and adjusting)}$$

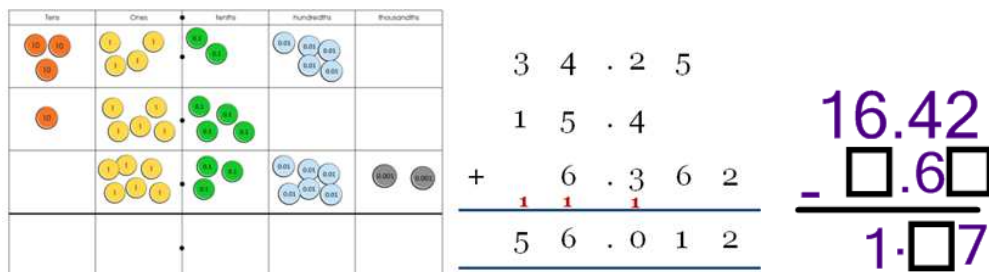
$$2.5 + 2.6 = \text{double } 2.5 \text{ and add } 0.1 \text{ (near doubles)}$$

When subtracting, pupils understand that when the two numbers are increased by the same amount, the difference does not change.

e.g. $258 - 149 = 259 - 150$

$$43,088 - 42,059 = 43,089 - 42,060$$

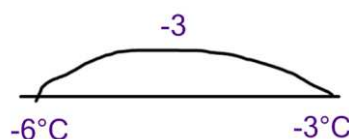
8.1 Add and subtract large numbers (including decimals) using the columnar method



8.2 Add and subtract positive and negative integers using the number line

Negative numbers will still be used within a context, still using the key concept of zero to bridge.

The temperature yesterday was -3°C and it has dropped by a further 3°C today. What is the temperature today?



SECTION B

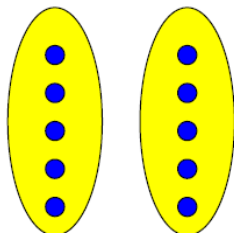
MULTIPLICATION AND DIVISION STRATEGIES (BY YEAR GROUP)

Year 1

Multiplication and division will be taught alongside each other to develop confidence and fluency with whole numbers. Pupils are beginning to develop their understanding of multiplication and division through grouping and sharing small quantities. They begin to build on their knowledge of doubling and halving, making links with fractions. In Year 1, pupils do not need to record statements using \times , \div or fraction symbols. Pupils make connections between arrays, number patterns and counting in 2s, 5s and 10s. Pupils will use a range of concrete materials to develop their understanding (e.g. cubes, counters, Dienes, Numicon etc.)

9.1 Double and halve numbers and quantities though grouping and sharing

Pupils will understand the relationship between doubling and halving and will use similar diagrams to represent both of these *i.e.* *half of 10 is 5, double 5 is 10*. Both concepts will be taught alongside each other for better conceptual understanding.

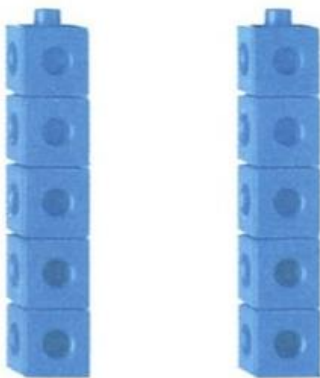
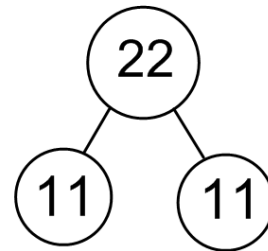


'2 groups of 5'

'How many altogether?'

' $5 + 5 = 10$ '

Double five is ten



'Double 5 is ten and half of 10 is 5'

It will be useful to teach pupils the concept of counting in twos, fives and tens alongside or before looking at multiplication and division. This will help to reinforce the link between counting in equal groups and equal sharing.



'Six pairs of socks.
How many socks altogether? 2, 4, 6, 8, 10, 12'

9.2 Solve multiplication and division using concrete and pictorial followed by arrays.

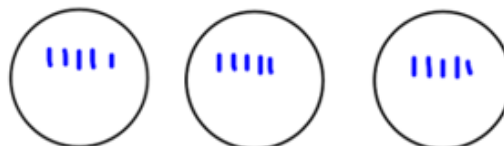
Groups of, lots of, shared between, equal groups



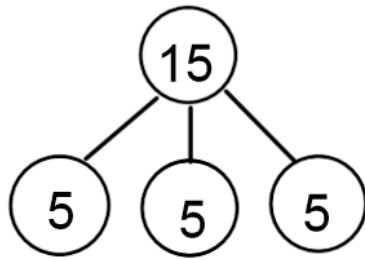
2 groups of 4
Two lots of four



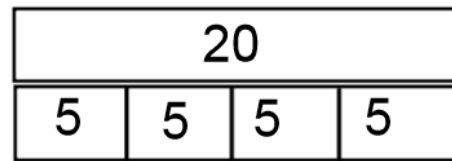
Pupils will be exposed to a range of pictorial representations.



15 shared into 3 equal groups.
There are 5 in each group.

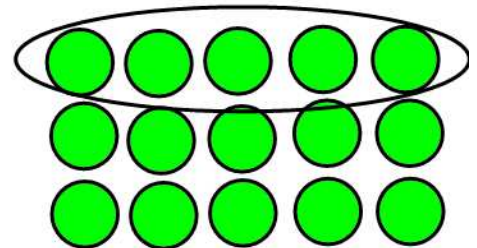


15 is the whole.
15 is shared into 3 equal groups.
There are 5 in each group.



20 is the whole.
There are 4 equal groups.
One quarter of 20 is 5.

When pupils begin to use arrays, they will start by identifying the number of rows as the number of equal groups and state how many are in each of the equal groups.

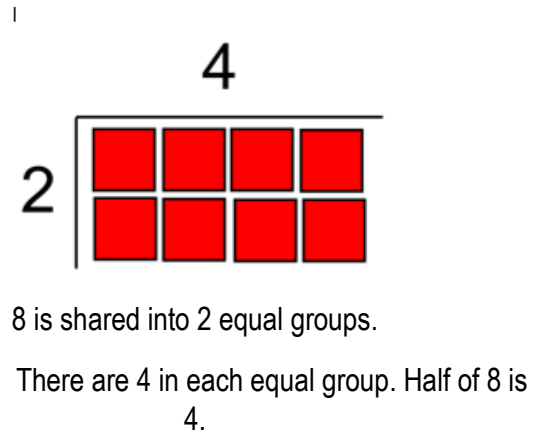
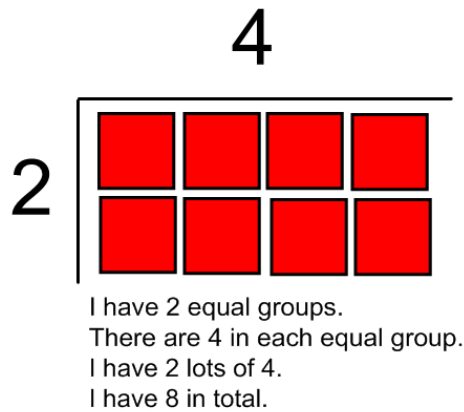


Arrays will be used for both multiplication and division.

Three equal groups of 5 is the same as 15.

15 shared into 3 equal groups. There are 5 in each group.

Pupils will then move onto labelling the arrays as shown below and make mathematical statements. This leads into short division in future years.

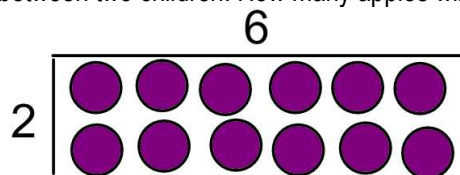


Pupils will be aware of the two concepts within division: sharing and grouping. This would be best introduced in context:

9.3 Sharing

Pupils will share a set of objects, equally:

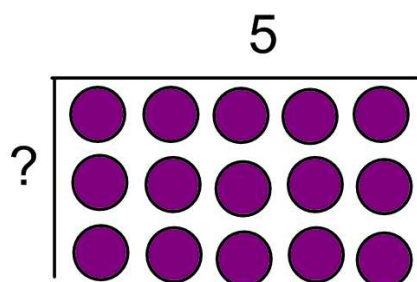
Share 12 apples equally between two children. How many apples will they each get? (Sharing)



There are 2 equal groups
There are 6 in each equal group

9.4 Grouping

There are 15 biscuits in a pack. If we put five biscuits on each plate, how many plates will we need?



Note: Teachers should distribute 5 counters across the top to make one row and state that this is one plate of 5. We need to find how many plates we can have and therefore how many rows/groups of 5.

I have 15 in total. If I count in groups of 5, I can make 3 equal groups.

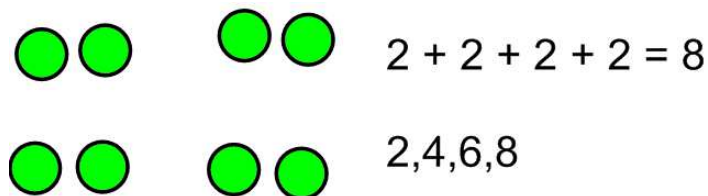
Year 2

By the end of Year 2, pupils will be able to confidently recall 2, 5 and 10 times tables and connect them to each other. Throughout the year, pupils will make links between these multiplication and division facts.

10.1 Solve problems involving \times and \div , using repeated addition and inverse relationships

Pupils will build on their knowledge of 'groups of' using pictorial and concrete resources and link it to repeated addition in context.

Kofi ate 2 apples every day for 4 days. How many apples did he eat altogether?

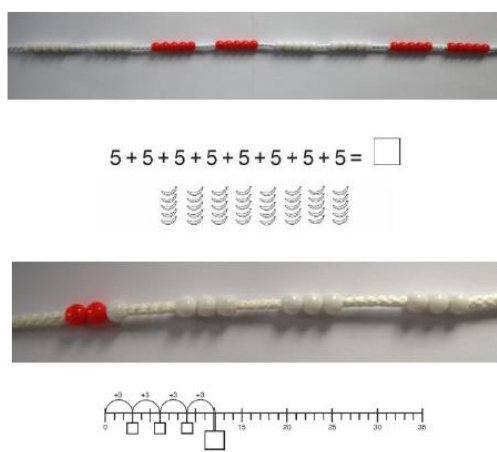


'There are 4 equal groups of 2.'

'There are 4 lots of 2.'

Pupils will then move onto representing this as a multiplication calculation: $4 \times 2 = 8$

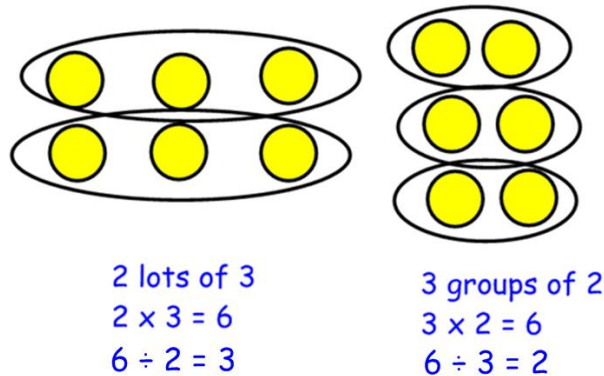
A range of resources and diagrams will be used to support their visual understanding.



10.2 Explore the commutative law of multiplication and link to related division facts.

Once pupils are secure with 2,5, and 10 times tables, they begin to develop links with other multiplication tables, especially the 3 times tables (linking to counting in 3s and thirds)

$$5 \times 3 = 15, \quad 3 \times 5 = 15, \quad 15 \div 5 = 3, \quad 15 \div 3 = 5$$

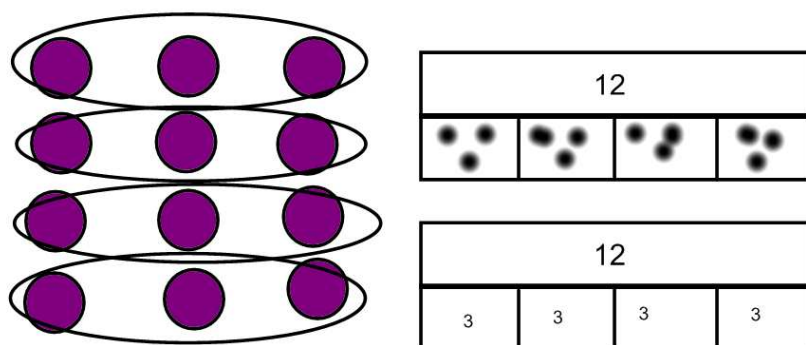


10.3 Sharing and grouping

Pupils will develop their understanding of sharing and grouping using appropriate diagrams and understanding the difference between these.

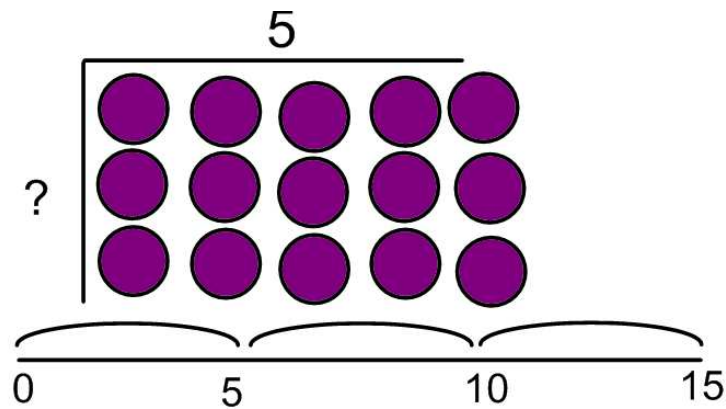
Sharing:

There are 12 children. The teacher puts them into 4 equal groups. How many children are there in each group?



Grouping:

There are 15 children. The teacher puts them into groups of 5. How many groups are there?



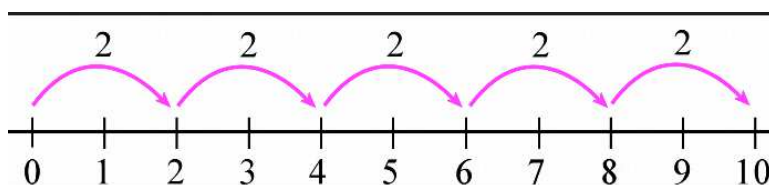
Counting on and back could also be done using fingers to count jumps moving onto rapid recall.

There are 15 altogether

There are 5 in each equal group

There are 3 groups of 5

Children could also use number lines to count in multiples of given number to find how many groups there are.



How many groups of two are there in 10?

$$10 \div 2 = 5$$

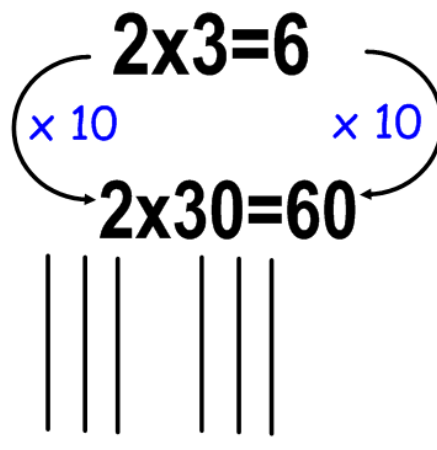
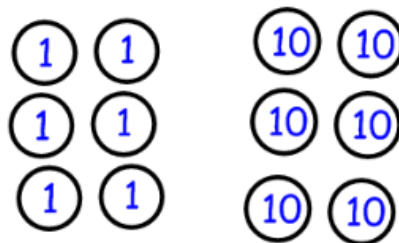
Year 3

In Year 3, pupils count in multiples of 4, 8 and 50 from zero and they need to recall and use 2, 3, 4, 8 and 10 times tables, applying these to division. Pupils build on their understanding of division and multiplication and use this to derive other facts.

11.1 Deriving related facts using times tables that they know

Using equipment such as place value counters and Dienes

$$3 \times 2 = 6 \text{ so } 30 \times 2 = 60$$



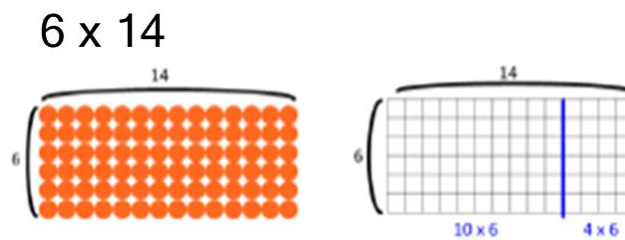
$$6 \div 3 = 2$$

$$60 \div 3 = 20$$

6 shared into 3 equal groups gives 2 in each equal group.
60 shared into 3 equal groups gives 20 in each equal group.

11.2 Develop efficient methods for multiplication of one- and two-digit numbers progressing to formal written methods

In Year 3, pupils work with teen numbers multiplied by single digit numbers within known times tables. They use their knowledge of place value to support their understanding of formal methods.



$$6 \times 14 = 6 \times 10 + 4 \times 6$$

17×3

$10 + 7$

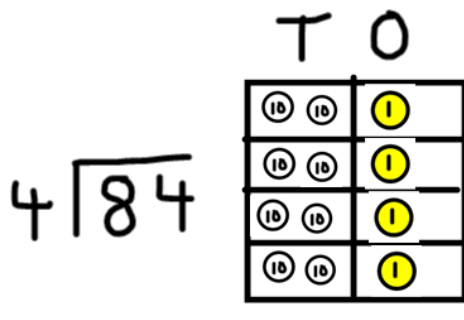
$$\begin{array}{r} 17 \\ \times 3 \\ \hline 51 \\ \hline \end{array}$$

$7 \times 3 = 21$
 $10 \times 3 = 30$

"7 times 3 is equal to 21.
 We have 1 one and 2 tens so the 2 goes into the tens column.
 $10 \times 3 = 30$.
 We have three tens plus the two tens is the same as five tens."

11.3 Short division for two-digit numbers

When using counters to model short division, pupils should be sharing rather than grouping (e.g. 84 shared into 4 groups.)



"8 tens shared into 4 equal groups gives us 2 tens. 4 ones shared into 4 equal groups gives us 1 one. 2 tens and 1 one is equal to 21."

Year 4

In Year 4, children learn the 6, 7, 9, 11 and 12 times tables facts so that by the end of Year 4, they are fluent in all times tables up to 12×12 . They also need to know how to multiply and divide by 1 and 0.

12.1 Multiplying three numbers together

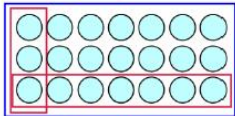
$$2 \times 3 \times 100 = 6 \times 100$$

$$5 \times 6 \times 2 = 5 \times 2 \times 6 = 10 \times 6$$

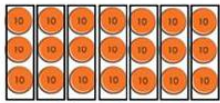
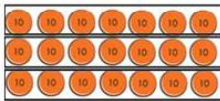
12.2 Deriving known facts for multiplication and division of larger numbers

Pupils will continue using the same materials and resources as in Year 3 for derived facts, using larger numbers ($600 \div 3 = 200$ can be derived from $2 \times 3 = 6$)



$21 \div 3 = 7$



$21 \div 7 = 3$

210 \div 7 = 30
 210 \div 3 = 70
 210 \div 30 = 7
 e.g. 210 \div 70 = 3

2100 \div 7 = 300
 2100 \div 3 = 700
 2100 \div 300 = 7
 2100 \div 700 = 3

12.3 Develop efficient mental methods using associative and commutative laws

Four pots each containing two flowers which each have seven petals. How many petals in total?



$$(4 \times 2) \times 7 \text{ or } 4 \times (2 \times 7)$$

For two-digit and one-digit numbers:

$$48 \times 5 = 8 \times 6 \times 5 = (8 \times 5) \times 6$$

$$6 \times 34$$



$$6 \times 30 + 6 \times 4$$



12.4 Divide two-digit numbers by 10 and 100 (identify the value of the digits in the answer as ones, tenths and hundredths)

Pupils should be secure with the concept of one tenth and understand that these arise by dividing a one digit number into ten equal parts. Lots of concrete and visual aids should be used to introduce this concept. Pupils should be able to place tenths on a number line in order to show their understanding of decimal tenths and hundredths as a parts of a whole.

12.5 Using place value counters and a model to show ten equal parts







$$12 \div 10$$

To divide ten by ten, we split ten ones into ten equal groups. This gives us one whole in each group.

Hundreds	Tens	Ones
		

12.7 Short division

For conceptual understanding, teachers should start by modelling using counters to physically share into the number of groups - this will support their understanding of regrouping/carrying and will link to previous array models. Pupils should use materials but not rely on them. In Year 4, divisions should always have exact answers (no remainders.)








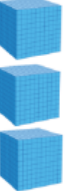






$3 \overline{) 3486}$	Th	H	T	O
			   	
			   	
			   	

Year 5

Pupils continue to build on mental methods for multiplication and division, applying this to larger numbers and decimals. In Year 5, pupils move on to multiplying and dividing abstract decimal numbers, building on their knowledge of decimals for measurement in Year 4. Pupils begin to interpret remainders in different ways.

13.1 Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000

In Year 5, pupils build on Year 4 strategies for multiplying and dividing by 10 and 100. They apply these to larger numbers and numbers involving decimals.

thousands 	hundreds 	tens 	ones 
			 3
		 3	
 3	 3		
			

$$30 \div 10 = 3$$

$$300 \div 100 = 3$$









$$3000 \div 1000 = 3$$

$$300 \div 10 = 30$$

$$3000 \div 100 = 30$$

$$3000 \div 10 = 300$$

$$102.14 \times 10 = 1021.4$$

Thousands	Hundreds	Tens	Ones	•	Tenths	Hundredths
						
						

13.2 Multiply numbers up to 4 digits by a one-digit number using a formal written method

See guidance for Year 4 with increased digits.

13.3 Multiply numbers to 4 digits by a two-digit number using formal written method [long multiplication]

$$\begin{array}{r}
 243 \\
 \times 12 \\
 \hline
 486 \\
 2430 \\
 \hline
 \end{array}$$

Thousands	Hundreds	Tens	Ones
	200 100 100	40 30 10	6 3 3
	200	40 30 10	6 3 3
2000 1000	400 300 100	80 60 20	

243×2

 243×10

243×2

 243×10

$$\begin{array}{r}
 34 \\
 \times 12 \\
 \hline
 68 \\
 340 \\
 \hline
 408
 \end{array}$$

13.4 Divide numbers to 4 digits by a one-digit number using the formal written method [short division]

$8528 \div 4$

Sharing

$$\begin{array}{r}
 2132 \\
 4 \overline{) 8528}
 \end{array}$$

Thousands	Hundreds	Tens	Ones
1000 1000	100	10 10 10	1 1
1000 1000	100	10 10 10	1 1
1000 1000	100	10 10 10	1 1
1000 1000	100	10 10 10	1 1

8 thousands shared into 4 equal groups
 5 hundreds shared into 4 equal groups
 1 hundred regrouped for 10 tens
 12 tens shared into 4 equal groups
 8 ones shared into 4 equal groups.

13.5 Short division interpreting remainders as fractions then common decimal equivalents

496 ÷ 11 becomes

$$\begin{array}{r} 45 \text{ r } 1 \\ 11 \overline{) 496} \end{array}$$

Answer: $45 \frac{1}{11}$

Leading into decimal equivalents of common fractions or fractions with a denominator of a multiple or a factor of 10 or 25 (e.g. $1/5 = 2/10 = 5/25$)

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$$

$$86 \frac{2}{5} = 86 \frac{4}{10} = 86.4$$

Year 6

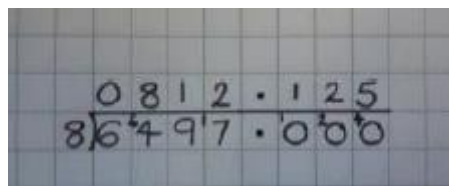
Pupils to continue to use models and images from previous years, working towards fluency without materials.

14.1 Multiplication of numbers with up to two decimal places by one digit and two-digit whole numbers

$$\begin{array}{r} \overset{2}{1}.\overset{6}{28} \\ \times 8 \\ \hline 10.24 \end{array}$$

When modelling this method, ensure the correct use of vocabulary: ones, tenths, hundredths and ensure pupils understand that we are multiplying each part of the 1.28 by 8 because we need 8 lots of each part. This is why the 8 does not need to line up with the ones because it represents 8 groups not 8 ones.

14.2 Short division including interpreting remainders as decimals



$$\begin{array}{r} 0.8125 \\ 8 \overline{) 6497.000} \\ \underline{64} \\ 97 \\ \underline{96} \\ 17 \\ \underline{16} \\ 170 \\ \underline{160} \\ 100 \\ \underline{96} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

14.3 New learning: long division

$$\begin{array}{r} 34 \\ 12 \overline{) 408} \\ \underline{36} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

Including interpreting remainders as decimals (builds on known decimal equivalences from Y5)

432 ÷ 15 becomes

$$\begin{array}{r}
 28.8 \\
 15 \overline{) 432.0} \\
 \underline{30} \\
 132 \\
 \underline{120} \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

Answer: 28.8